Energy interactions in homogeneously sheared magnetohydrodynamic flows

Diane Collard[1], Divya Sri Praturi[2], Sharath S. Girimaji[2]
[1]Chemical Engineering, Kansas State University
[2]Aerospace Engineering, Texas A&M University

69th Annual meeting of American Physical Society, Division of Fluid Dynamics

Outline

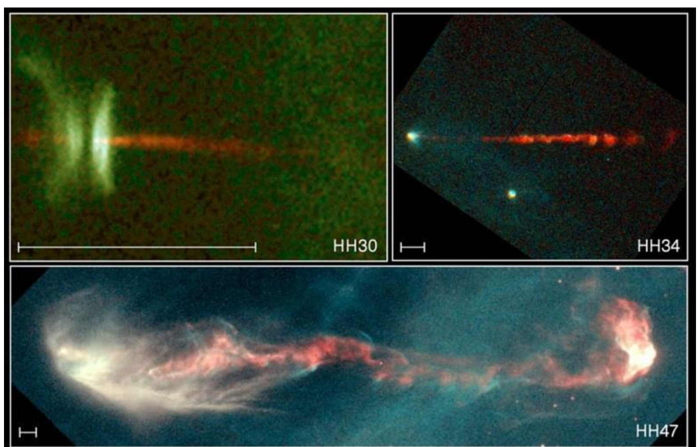
- Motivation
- Objectives
- Approach
- Simulation setup
- Results
- Conclusions

Motivation

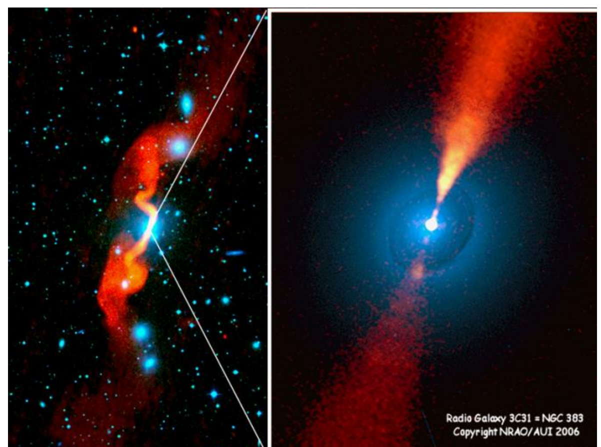
Kansas State University College of Engineering

Aerospace Engineering Texas A&M University

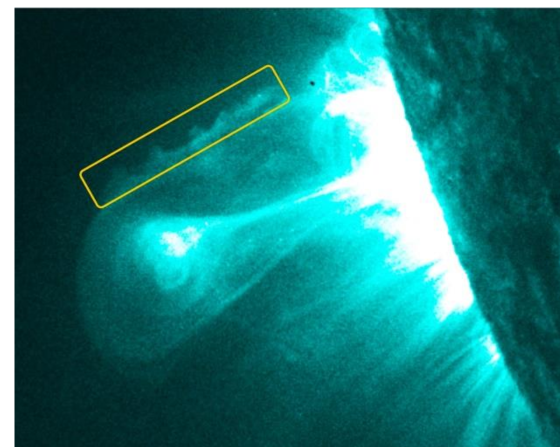
- Plasma is ubiquitous – constitutes ~99% of baryonic matter
  - Widely observed in nature and engineering
- Magnetic field is generally known to stabilize instabilities
  - Kelvin-Helmholtz or Richtmeyer-Meskov instability



Jets from young stars
Photo credit: C. Burrows (ST ScI), J. Hester (ASU), J.Morse (ST ScI), NASA



Active galactic nuclei of radio galaxy 3C31
Photo credit: NRAO/AUI 2006



Kelvin-Helmholtz instability in a coronal mass ejection
Photo credit: NASA AIA/SDO

Objectives

- Investigate the evolution of velocity and magnetic fields as a function of
  - Magnetic field strength (B_0)
  - Perturbation wavevector orientation (β)
- Investigate energy exchange between perturbation velocity and magnetic fields

Theory

- Considering perturbations in shearnormal plane:

$$\kappa_1(t) = \kappa_0 \cos\beta, \kappa_2(t) = -\kappa_0 \cos\beta\, St,$$
$$\kappa_3(t) = \kappa_0 \sin\beta$$

κ_0 is the initial wavevector magnitude

- Dimensionless quantities using reference distance, time, velocity:

$$\kappa_{ref} = \kappa_0; \;\; t_{ref} = \frac{1}{S}; \;\; u_{ref} = V_A = \frac{B_0}{\sqrt{\bar{\rho}\mu_0}}$$

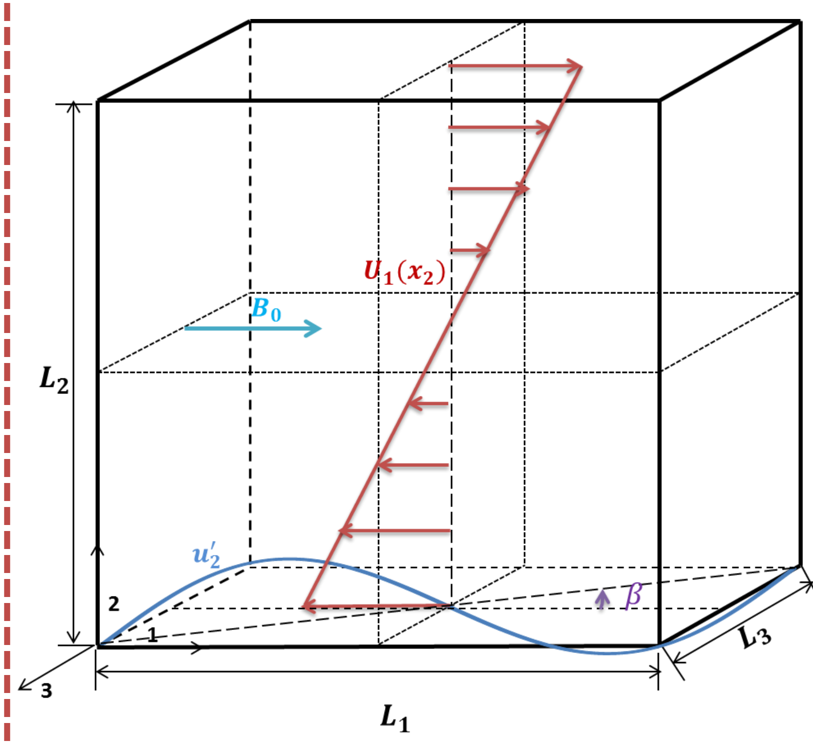
- Dimensionless governing equations in spectral space:

$$\frac{d\hat{u}_i^*}{d\tau} = \hat{u}_2^* \left(-\delta_{i1} + \frac{2\cos\beta\kappa_i^*}{1 + \cos^2\beta\,\tau^2} \right) + iR_A^* \hat{B}_i^*$$

$$\frac{d\hat{B}_i^*}{d\tau} = \hat{B}_2^* \delta_{i1} + iR_A^* \hat{u}_i^*$$

$$R_A^* = \frac{\kappa_0 V_A \cos\beta}{S} = R_A \cos\beta$$

Where, $i = 1 - 3$



Problem Setup

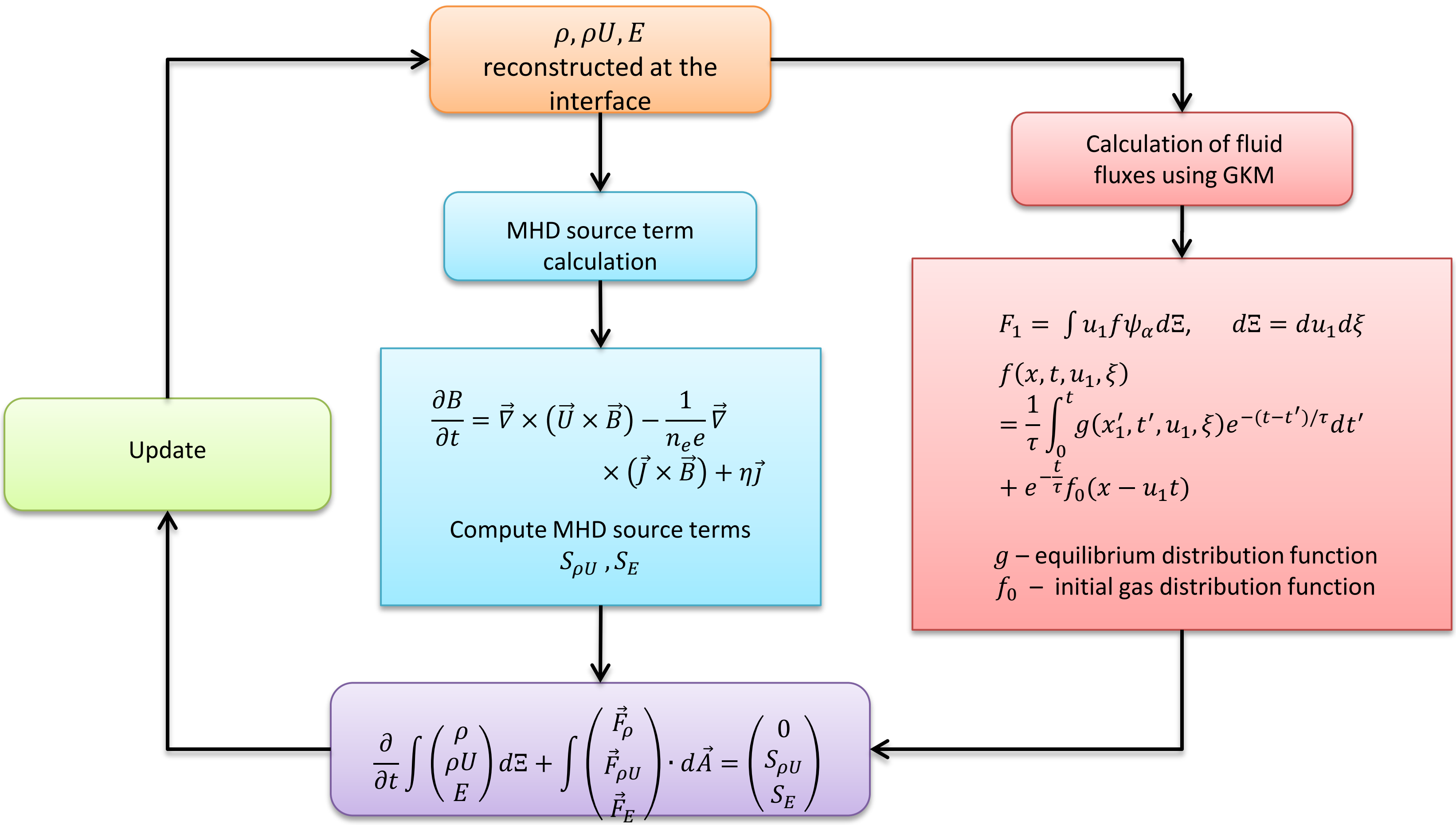
Theory

- The evolution of velocity and magnetic fields decided by R_A^*
  - R_A^*: Ratio of shear $\left(\tau_S = \frac{1}{S}\right)$ and magnetic time scales $\left(\tau_B = \frac{1}{\kappa_0 V_A \cos\beta}\right)$
  - Magnetic frequency: $\frac{1}{\tau_B} \propto \cos\beta$
- For a given V_A, S, κ_0:
  - Streamwise perturbations ($\beta = 0^\circ$): R_A^* is maximum
    - Highest harmonic exchange between $\hat{u}_i^*$ and $\hat{B}_i^*$ $\Rightarrow$ equi-partition between kinetic and magnetic energies can be observed
  - Spanwise perturbations ($\beta = 90^\circ$): $R_A^* = 0$
    - No exchange between $\hat{u}_i^*$ and $\hat{B}_i^*$ $\Rightarrow$ no evolution of $\hat{B}_i^*$
    - $\hat{u}_i^*$ equations are pressure-released
    - Kinetic energy grows quadratically unaffected by magnetic field strength
  - Intermediate orientations ($\beta = 30^\circ, 60^\circ$):
    - Mixed behavior depending on the orientation

Numerical Scheme

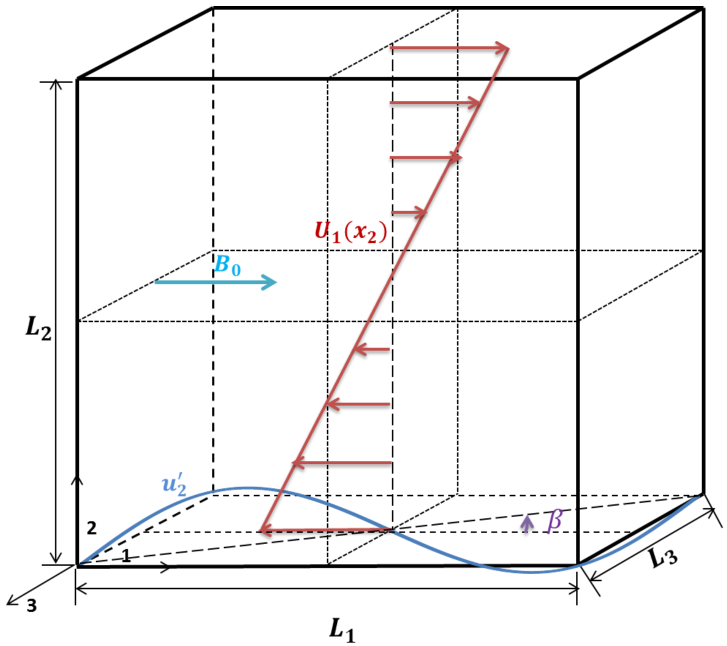
Kansas State University College of Engineering
Aerospace Engineering Texas A&M University

- Magneto-Gas Kinetic Method[1,2] (MGKM) solves fluid equations with the simplified Boltzmann equation and the magnetic field equations *separately.*



[1]Xu, K., Journal of Computational Physics 171, 289-335 (2001)
[2]Araya, D.B. et al. , ASME, Vol. 137, 081302-(1-11), Aug., 2015

Simulation setup

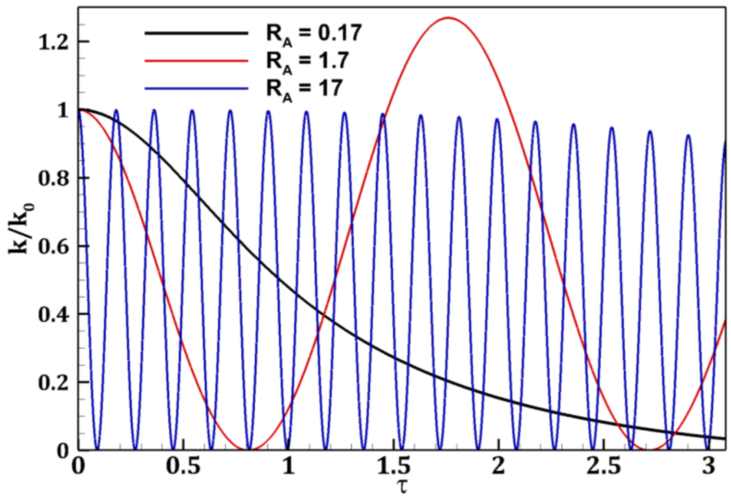


- MHD homogeneous shear simulation conditions
  - Temperature = 300 K, Density = 1 kg/m^3
  - Reynolds number = 770
  - Magnetic Reynolds Number = 195
  - Gradient Mach number, $\mathrm{M_g} = 0.03$
- Boundary conditions:
  - 1 and 3 planes: periodic boundaries
  - 2 plane: shear periodic boundary

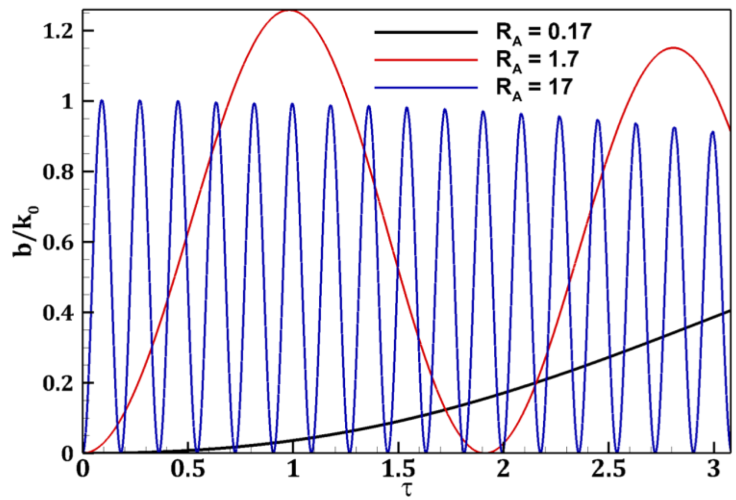
Magnetic field strength (B_0)	$V_A = \frac{B_0}{\sqrt{\mu_0 \rho}}$	Perturbation orientation (β)	$R_A = \frac{V_A \kappa_0}{S}$	$R_A^*(\beta = 0°, 30°, 60°, 90°)$
0.0003 T	0.26m/s	$0°, 30°, 60°, 90°$	0.17	0.17, 0.14, 0.085,0
0.003 T	2.6m/s	$0°, 30°, 60°, 90°$	1.7	1.7, 1.4, 0.85, 0
0.03 T	26m/s	$0°, 30°, 60°, 90°$	17	17, 14, 8.5, 0

Simulation parameters

Effect of magnetic field strength at $\beta = 0°$



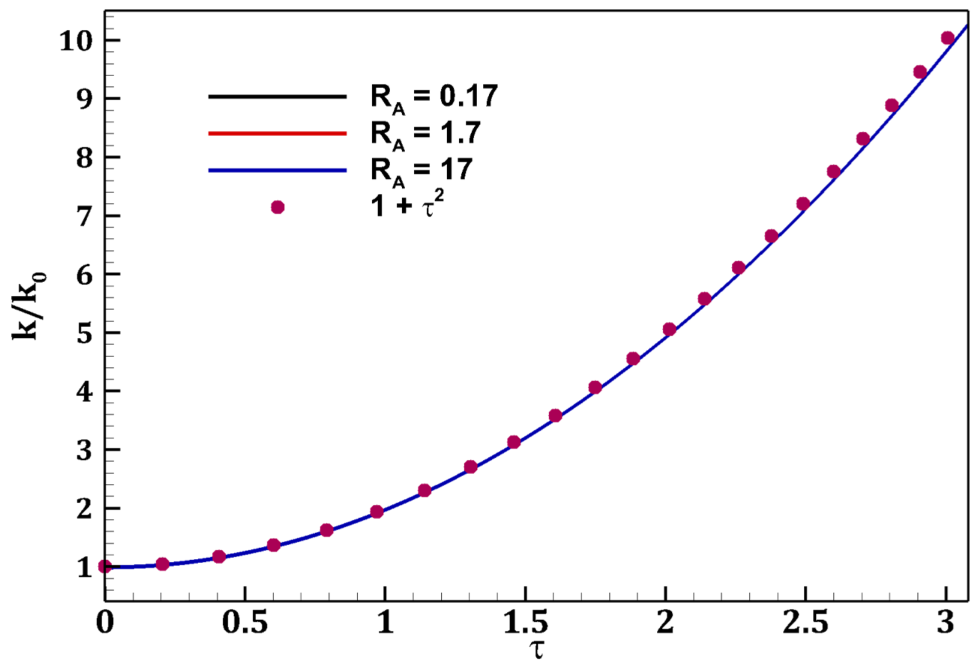
Evolution of perturbation kinetic energy $(\boldsymbol{k})$ with respect to initial kinetic energy $(\boldsymbol{k_0})$



Evolution of perturbation magnetic energy $(\boldsymbol{b})$ with respect to initial kinetic energy $(\boldsymbol{k_0})$

- As R_A increases, wave-like behavior increases
- $R_A = 0.17$:
  - Kinetic energy monotonically decays to zero ⇒Transfer to mean kinetic energy via the action of pressure
  - Magnetic energy grows monotonically
- $R_A = 1.7$:
  - Solution exhibits wave-like behavior and overshoots the initial kinetic energy ⇒Conversion to perturbation kinetic energy from mean energy
- $R_A = 17$, solution oscillates and decays gradually

Effect of magnetic field strength at $\beta = 90°$



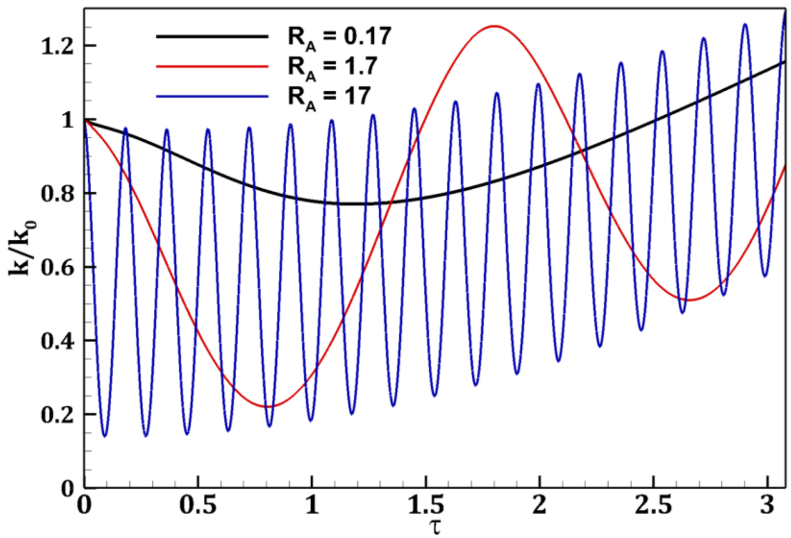
Evolution of perturbation kinetic energy (k) with respect to initial kinetic energy (k_0)

- Kinetic energy grows quadratically at all values of R_A
- The evolution matches the pressure-released Burgers evolution given by Simone et al.

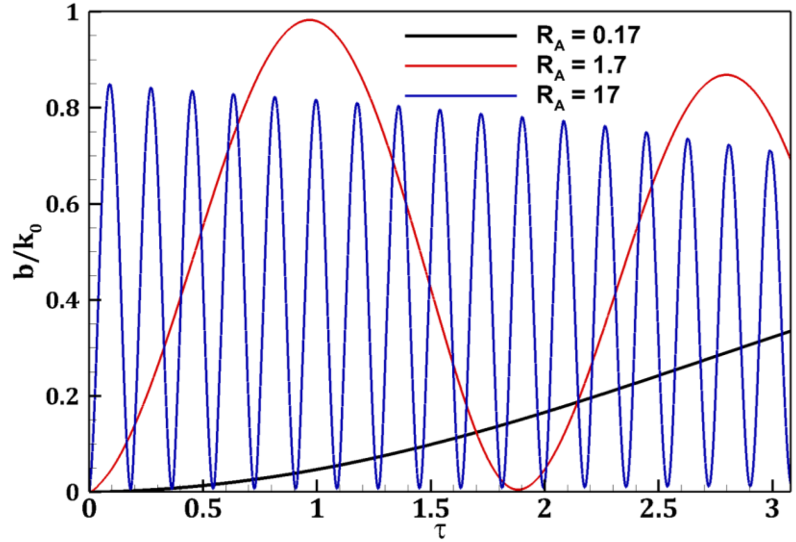
$$\frac{k}{k_0} = 1 + \tau^2$$

Effect of magnetic field strength at $\beta = 30°$

AEROSPACE ENGINEERING
TEXAS A&M UNIVERSITY



Evolution of perturbation kinetic energy (k) with respect to initial kinetic energy (k_0)

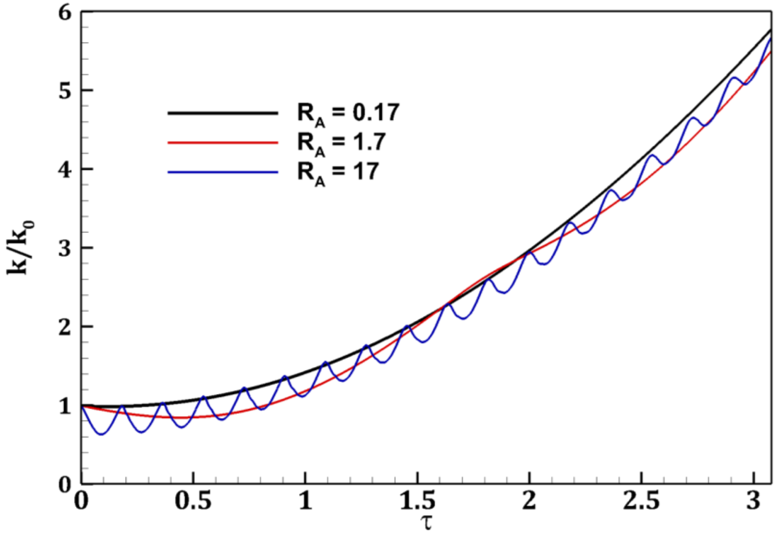


Evolution of perturbation magnetic energy (b) with respect to initial kinetic energy (k_0)

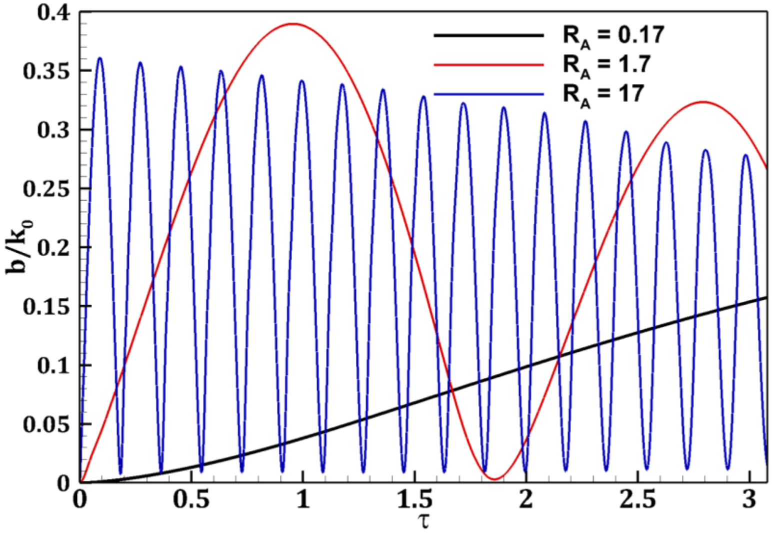
- At all R_A, kinetic energy decreases initially, but increases at later times ⇒ spanwise behavior dominates at later times
- Magnetic energy evolution similar to $\beta = 30°$, except at lower magnitudes

Effect of magnetic field strength at $\beta = 60°$

AEROSPACE ENGINEERING
TEXAS A&M UNIVERSITY



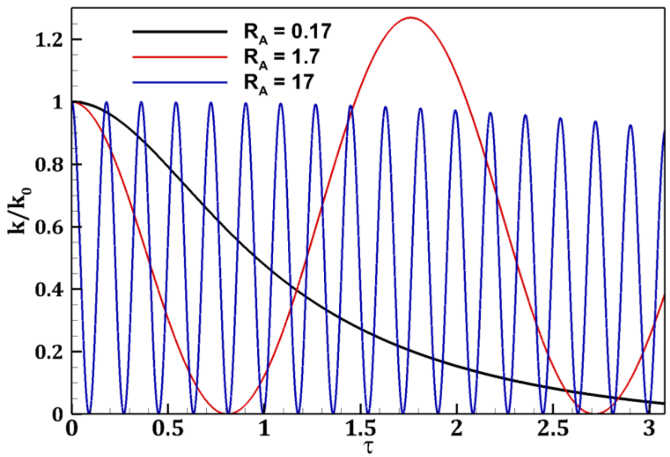
Evolution of perturbation kinetic energy $(\boldsymbol{k})$ with respect to initial kinetic energy $(\boldsymbol{k_0})$



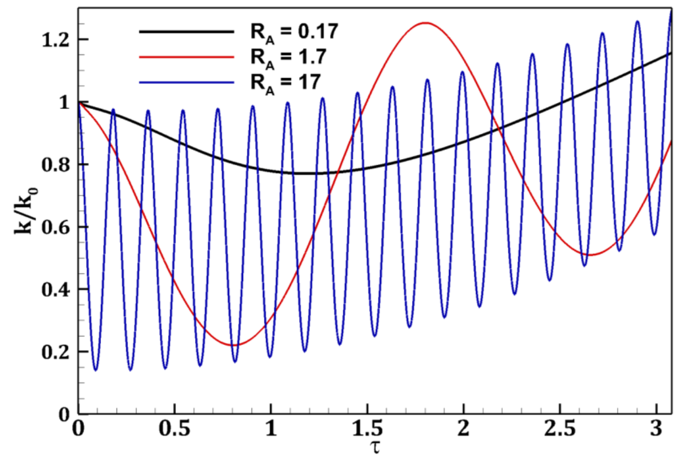
Evolution of perturbation magnetic energy $(\boldsymbol{b})$ with respect to initial kinetic energy $(\boldsymbol{k_0})$

- At all R_A, kinetic energy exhibits similar growth; oscillations about $R_A = 0.17$ evolution decrease in magnitude
- Magnetic energy plots similar to $\beta = 0°, 30°$; except at lower magnitude
- Predominantly spanwise behavior

The effect of orientation (β)

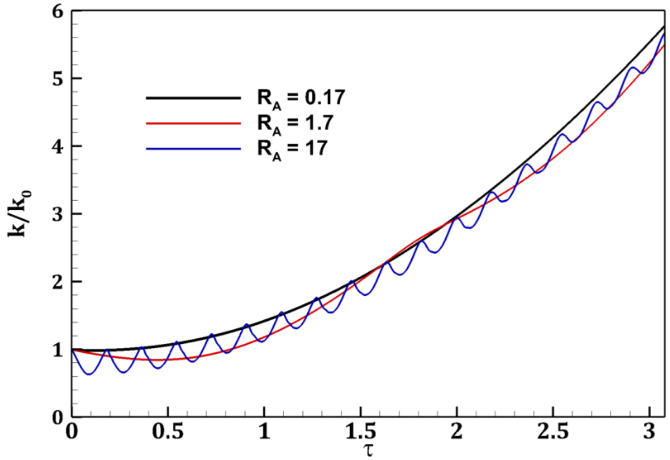


$\beta = 0°$

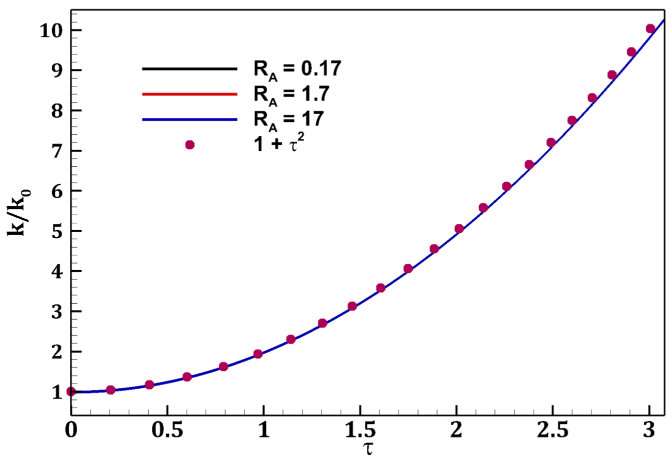


$\beta = 30°$

As β increases, the effect of magnetic field decreases



$\beta = 60°$



$\beta = 90°$

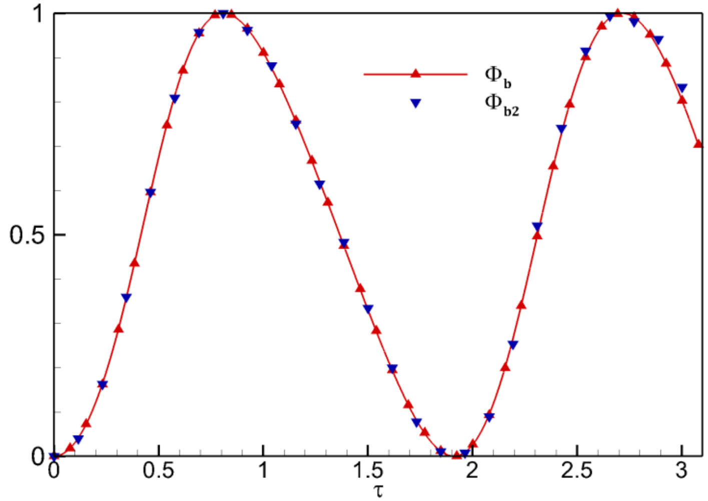
Harmonic energy exchanges

- Kinetic and magnetic energies exhibit similar magnitudes at $\beta = 0°$
- Equi-partition potentials defined as:

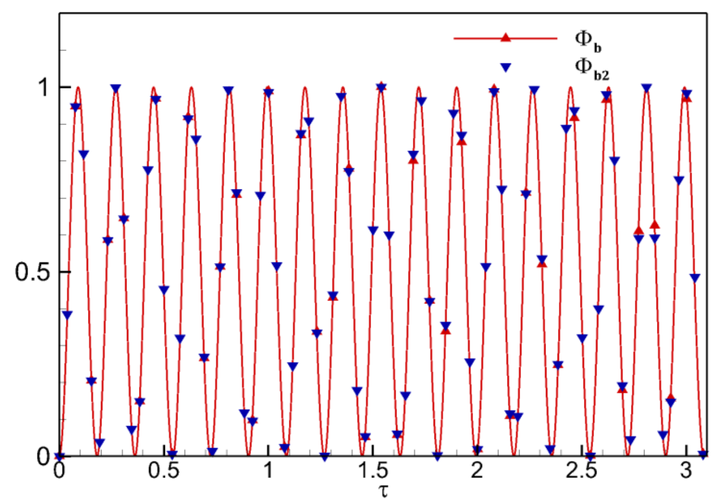
$$\Phi_b = \frac{b}{k + b}$$

$$\Phi_{b2} = \frac{b_2}{k_2 + b_2}$$

Where, k, k_2 are total, 2-component kinetic energies, and b, b_2 are total, 2-component magnetic energies



$R_A = 1.7$



$R_A = 17$

- Oscillates about 0.5 with frequency of π/R_A
  - Equi-partition of kinetic and magnetic energies and their 2-component

Conclusions

- $R_A^* = \frac{V_A \kappa_0 \cos\beta}{S}$ characterizes the effect of magnetic field strength and wavevector orientation

- As magnetic field strength increases, R_A^* increases :
  - Wave-like behavior results

- As β increases from $0°$ to $90°$, R_A^* decreases:
  - $\beta = 0°$: highest harmonic exchange, equi-partition at $R_A = 1.7, 17$
  - $\beta = 90°$: no exchange, pressure-released behavior at all magnetic field strengths
  - As β varies from $0°$ to $90°$, the effect of magnetic field decreases

Kansas State University College of Engineering

Aerospace Engineering Texas A&M University

Thank you!

Questions?